Name ______________________________

C. The "Birthday Paradox" Problem

Special Instructions: Please read the background information in the handout before attempting to complete this worksheet.

Goal: to calculate the probability that any two people in a randomly chosen group of people will have the same birthday.

We imagine a group of people, designated A, B, C, D, and so forth, as well as a large calendar where each date is represented by a box. We assume that there are 365 days in a year.

1. Begin with Person A. Imagine that Person A marks an X on the date of his or her birthday. Then person B goes to the calendar to mark an X on the date of his or her birthday.

a. What is the probability of the event that Person A has the same birthday as Person B? Equivalently, what is the probability that Person B will mark an X on the same box as Person A?

b. What is the probability of the event that Person B has a birthday on a different day than Person A? Equivalently, what is the probability that Person B will mark an X in a different box than Person A?

2. Suppose Person C comes along.

a. Assuming that Persons A and B have different birthdays, how many boxes have X marks in them?

b. When Person C marks an X on the date of his or her birthday, what is the probability of the event that Person C's X will fall in a box which already contains an X?

c. What is the probability of the event that Person C will mark a box that does not already contain an X?

d. Combine the results of 1b and 2c to determine the probability of the event that the 3 X's of Persons A, B, and C will all be in different boxes. Note that there are two probabilities that must be multiplied together. The first probability is the probability that Person B has a different birthday than Person A, and the second probability is the probability that Person C has a different birthday than Persons A and B (assuming that A and B do indeed have different birthdays).

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e. Using the principle for determining the probability of the complement of an event, determine the probability of the event that in a group of three people, A, B, and C, two or more have the same birthday.

3. Continue the process described in problems 1 and 2. Suppose Person D comes along.

a. Assuming that Persons A, B, and C have different birthdays and have marked X's in three different boxes, what is the probability of the event that Person D will mark his or her X in one of the three boxes already marked?

b. What is the probability of the event that Person D will not mark his or her X in one of the three boxes already marked?

c. Combine the results of 2d and 3b to find the probability of the event that given four people, A, B, C, and D, no one will share a birthday.

d. Using the principle for determining the probability of the complement of an event, determine the probability that in a group of four people, A, B, C, and D, two or more have the same birthday.

4. Considering your answers to 1b, 2c, and 3c, write a mathematical expression describing the probability of the event that, in a group of k people, none share a birthday. (Here, assume that *k* can be any counting number. Do not assume that there must be 11 people because *k* is the 11th letter of the alphabet.)

5. Use the expression from Problem 4 and what you know about the probability of the complement of an event to express in terms of *k* the probability that in a group of *k* people, there are two or more people with the same birthday.

6. Use the expression from Problem 4 to numerically determine the chances that a group of people do not have the same birthday for the following size groups of people: 3, 5, 10, 15, and 20.

BONUS:

7. Determine to two decimal places the probability of the event that among 23 people chosen at random, there are no people who share the same birthday. What does this answer tell us about the probability of the event that in a group of 23 randomly chosen people, two do share the same birthday?

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